7.3 Direct update of breeding values

As a basic rule for direct updating 3 types of information on an animal breeding value exist; the parents, own performance and the offspring. The pieces of information derived from half and full sibs as well as from grandparents are all included in the parent's breeding value. This is both a strength and a weakness in the direct updating method as the parents can only be included once. Therefore, new information on half and full sibs cannot be utilized. Usually this is unimportant, so it is an advantage in several cases not to bother about this information.

The information, gathered from the 3 different sources for the use of estimating the breeding value, has the following forms. The heritability is h^2 and common environmental variance c^2. The estimated breeding value on an absolute scale has the symbol A hat. The formulas for combining the indexes (measured as deviations from the average) from the 3 possible sources can be seen below.

Parents:

$$\hat{A} = (\hat{A}_{Dam} + \hat{A}_{Sire})/2 \quad , where \; I_1 = \hat{A} - \bar{A} \; and \; r^2_{AI_1} = (r^2_{AI,Dam} + r^2_{AI,Sire})/4$$

Own performance (n observations with mean value $\bar{P}_g$ and repeatability $r = h^2 + c^2$):

$$\hat{A} = \bar{A} + \frac{nh^2}{1+(n-1)(h^2+c^2)} (\bar{P}_g - \bar{P}) \quad ,where \; I_2 = \hat{A} - \bar{A} \; and \; r^2_{A,I_2} = \frac{nh^2}{1+(n-1)(h^2+c^2)}$$

Half or full sib offspring (n number of offspring with mean value $\bar{P}_g$):

$$\hat{A} = \bar{A} + \frac{.5nh^2}{1+(n-1)(ah^2+c^2)} (\bar{P}_g - \bar{P}) \quad ,where \; I_3 = \hat{A} - \bar{A} \; and \; r^2_{A,I_3} = \frac{.25nh^2}{1+(n-1)(ah^2+c^2)}$$

where a=.25 and .5 for half and full sib offspring, respectively.

The indices I_1, I_2 and I_3 can be combined in a new index I_c

$$I_c = b_x I_x + b_y I_y \quad where \; x=1,2,3 \; or \; c \; and \; y=1,2,3 \; or \; c \; and$$

$$b_x = \frac{1-r^2_{AI_y}}{1-r^2_{I_xI_y}} \; , \; b_y = \frac{1-r^2_{AI_x}}{1-r^2_{I_xI_y}} \; and \; r^2_{AI_c} = \frac{r^2_{A,I_x} + r^2_{AI_y} - 2r^2_{I_xI_y}}{1-r^2_{I_xI_y}}$$

There is only one condition, that is, the two indices must come from sources that only are related though the animal having its breeding value estimated.
This also implies that

$$r^2_{I_xI_y} = r^2_{A.I_x} \cdot r^2_{AI_y}$$

An example of this condition: an index has been made of full sib offspring and is included in the combination index. A new index of additional full sibs cannot be included without violating the rule, as they are related to the sibs in the first index, through both the father and the mother. Half sib offspring can, on the other hand, be added when new information arrives, since they are only related through the animal in question.